4.7 Identity and Inverse Matrices

Identity Matrix--square matrix that when multiplied by another matrix, it equals that same matrix.

$$A \cdot I = A \qquad I \cdot A = A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse matrices--are 2 square matrices whose product is the identity

A^{-1} -- "A inverse"

$$A \cdot A^{-1} = I$$

$$A^{-1} \cdot A = I$$

Are they inverses? yes

$X \cdot Y = I$

$Y \cdot X = I$

$X = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ 2x2 $\qquad Y = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ 2x2

$\begin{bmatrix} 1,1 & 1,2 \\ 2,1 & 2,2 \end{bmatrix}$

$\begin{bmatrix} 3(1)+(-2)(1) & 3(2)+(-2)(3) \\ -1(1)+1(1) & -1(2)+1(3) \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$Y = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \qquad X = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

$Y \cdot X \overset{?}{=} I$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1,1 & 1,2 \\ 2,1 & 2,2 \end{bmatrix}$

Are they inverses?

No

$P = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$ $Q = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

$PQ \overset{?}{=} I$

$QP \overset{?}{=} I$

$PQ = \begin{bmatrix} 1 & -13 \\ & \end{bmatrix}$

Finding the inverse.

If D = 0, there is no inverse.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find the inverse.

$\begin{bmatrix} 6 & 4 \\ -1 & 3 \end{bmatrix}$ -4 18

Inverse $\frac{1}{22}\begin{bmatrix} 3 & -4 \\ 1 & 6 \end{bmatrix}$

$\begin{bmatrix} \frac{3}{22} & -\frac{2}{11} \\ \frac{1}{22} & \frac{3}{11} \end{bmatrix}$

$H = \begin{bmatrix} -1 & 0 \\ 8 & -2 \end{bmatrix}$ 0 2

$H^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 0 \\ -8 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -4 & -\frac{1}{2} \end{bmatrix}$

$J = \begin{bmatrix} -4 & 6 \\ -2 & 3 \end{bmatrix}$ -12 -12

$D = 0$

No Inverse

Cryptography

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

13|38|24|49|44|107|19|57|22|53|17|39

$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$\begin{bmatrix} 13 & 38 \\ 24 & 49 \\ 44 & 107 \\ 19 & 57 \\ 22 & 53 \\ 17 & 39 \end{bmatrix}$ 6x2

A^{-1}

2x2

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

14|29|28|64|20|60|24|67|20|40|29|72|13|39|29|
72|5|11|25|50

HW
p199
10-13, 16-25